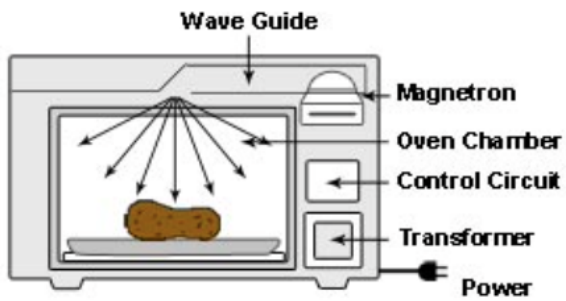
Problem 1: Microwave oven

At 2 GHz, the conductive of meat is on the order of 1 [S/m]. When a material is placed inside a microwave oven and the field is activated, the presence of the electromagnetic fields in the conducting material causes energy dissipation in the material in the form of heat.



a **Develop an expression for the time-average power per mm^3 dissipated in a material of conductivity σ if the peak electric field in the material is E_0.**
Consider a small volume of the meat as a box with length d and cross sectional area A. Along the direction of d, the $\overrightarrow{E}$ field will create a voltage difference across the box

$$V = Ed \tag{1}$$

The conduction current through the cross sectional area is

$$I = JA = \sigma EA \tag{2}$$

Thus the power is given by

$$P = IV = Ed \cdot \sigma EA = \sigma E^2(Ad) = \sigma E^2 v \tag{3}$$

Where $v = Ad$, and is the volume of the box.
We want Power per mm^3 so we set $v = (10^{-3})^3 \to 10^{-9} \quad [m^3] = [mm^3]$

$$\frac{P}{v} = \sigma E^2 \times 10^{-9} \qquad \left[\frac{W}{mm^3}\right] \tag{4}$$

Next we assume the microwaves are time harmonic signals, i.e.

$$E = E_0 \cos(\omega t) \tag{5}$$

We can take the time average power of time harmonic signals as

$$P_{av} = \left[\frac{1}{T}\int_o^T \sigma E_0^2 \cos^2(\omega t) dt\right] \times 10^{-9} \qquad \left[\frac{W}{mm^3}\right] \tag{6}$$

Finally giving us

$$P_{av} = \frac{1}{2}\sigma E_0^2 \times 10^{-9} \quad \left[\frac{W}{mm^3}\right] \tag{7}$$

b **Evaluate the result for an electric field** $E_0 = 4 \times 10^4 \quad \left[\frac{V}{m}\right]$**.**
We can simply plug our values into the equation above for

$$\begin{aligned} P_{av} &= \frac{1}{2}(1\frac{S}{m})(4 \times 10^4 \frac{V}{m})^2 \times 10^{-9} \quad \left[\frac{W}{mm^3}\right] \\ &= 0.8 \quad \left[\frac{W}{mm^3}\right] \end{aligned} \tag{8}$$

c **Explain why it is not advisable to use metal in a microwave oven. (Bonus - what might be the advantage of using a metal in a microwave oven just below the pizza crust.)**

(a) Metals are highly reflective and reflecting microwaves can damage the transmitter inside the unit or cause extreme heat build up which can start a fire.

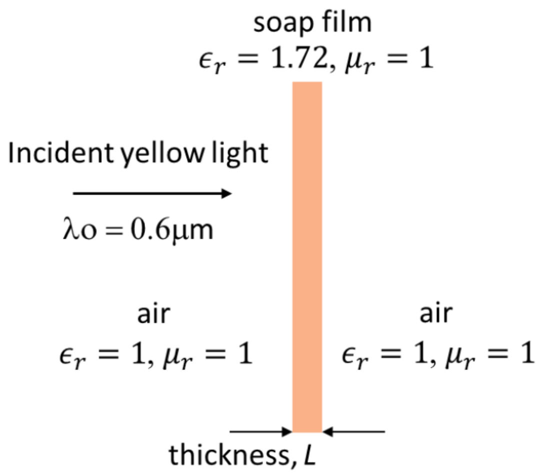
(b) For pointed metal objects (any non-perfect sphere), microwaves can cause a build-up of excess electrons at the tip which can then act as an electrode at high potential, creating plasma arcs (mini lightning) between the metal and the walls of the microwave oven, which can also start a fire.

Basically, you don't want to start a fire in your microwave.

Bonus: Using a metal foil in the microwave right below the pizza crust will cause reflections which redirect the microwaves back into the bottom of the pizza, resulting in higher localized heat at the crust and thus a crisper crust. We do not recommend doing this due to the aforementioned fire hazard.

Problem 3: Plane wave propagation through thin soap film

A thin film of soap is illuminated by yellow light at normal incidence of wavelength, $\lambda_0 = 0.6\mu m$ as shown in the figure below. (λ_0 is the wavelength of the yellow light in air or free space)



The film is surrounded on both sides by air (with free space parameters $\epsilon_r = 1, \mu_r = 1$) and can be treated as a planar perfect dielectric slab with parameters $\epsilon_r = 1.72, \mu_r = 1$. Obtain the thinnest film thickness that would produce the strongest reflection of the normally incident yellow light.

Because we are already comfortable with transmission line theory, it is both convenient and more intuitive to model this problem like a transmission line. We can imagine the air on the left side as a transmission line with the characteristic impedance of air $Z_{0|air}$, the soap film as another transmission line with a different characteristic impedance, $Z_{0|soap}$, and the air on the right side as a load with impedance $Z_L = Z_{0|air}$.

We are interested in the reflection at the air-soap film interface on the left side–this is analogous to the reflection coefficient between the characteristic impedance of the first transmission line and the input impedance of the soap film transmission line and air load.

$$\Gamma = \frac{Z_{in} - Z_{0|air}}{Z_{in} + Z_{0|air}} \tag{9}$$

Recall the equation for input impedance of a transmission line (remember $Z_L = Z_{0|air}$ and in this case, $Z_0 = Z_{0|soap}$):

$$Z_{in} = Z_{0|soap} \frac{Z_L + jZ_{0|soap} \tan(\beta l)}{Z_{0|soap} + jZ_L \tan(\beta l)} \tag{10}$$

By inspection, we see that when the tangent terms are 0, the equation simplifies to $Z_{in} = Z_L$. This happens when $\beta l = n\pi$, or when $l = \frac{n\lambda}{2}$, corresponding to an input reflection of 0. We want the opposite of this; we want to maximize the input reflection. This happens when the tangent terms are maximized, or when $\beta l = \frac{\pi}{2} + n\pi$.

$$\begin{aligned}\beta l &= \frac{\pi}{2} + n\pi \\ \frac{2\pi}{\lambda} l &= \frac{\pi}{2} + n\pi \\ l &= \frac{\lambda}{4} + \frac{n\lambda}{2}\end{aligned} \tag{11}$$

We can now solve for the wavelength of the yellow light in the soap film:

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{0.6\mu\text{m}}{\sqrt{1.72}} = 0.457 \quad \mu\text{m} \tag{12}$$

The thinnest film thickness that would result in the strongest reflection is therefore

$$l = \frac{\lambda}{4} = 0.114 \quad \mu\text{m} \tag{13}$$